

Examiners' Report Principal Examiner Feedback

January 2024

Pearson Edexcel International Advanced Level in Pure Mathematics P2 (WMA12) Paper 01

#### General

This paper proved to be a fair paper on the WMA12 content, and it was pleasing to see candidates were able to make attempts at all of the questions. Overall, marks were available to candidates of all abilities and the questions which proved to be most challenging were 5, 8 and 9. Time did not appear to be an issue for candidates.

Candidates should also be reminded to pay particular attention to any information at the start of questions in emboldened writing, which may indicate the degree to which a calculator can be used and any further emphasis on showing all stages of their working, in order to score all the available marks. The presentation of solutions is an important skill in being able to show the method, so solutions which could have come directly from a calculator are unlikely to be credited with full marks.

## **Report on individual questions=**

#### **Question 1**

This was an accessible question to start the paper and most candidates gained full marks, with very few failing to attempt it at all. The two main approaches were either using the remainder theorem or using algebraic long division. The remainder theorem was the most efficient approach, and this was also the most common.

Many candidates who used the remainder theorem did so without error and scored full marks. Conceptual misunderstandings were occasionally seen when trying to form an appropriate equation. The most common were using f(-2) or f(3) rather than f(2) or setting equal to -3 or 0, rather than 3. Arithmetic errors were also seen when attempting to solve the resulting linear equation.

Candidates who chose to use division gave themselves a more difficult task and tended to be less successful, with many giving up part way through the division. Most achieved  $ax^2$ , but then made errors either resulting in a quotient which did not meet the minimum requirements or leading to an incorrect expression for the remainder. In general, candidates who completed the division understood that they needed to set the remainder equal to 3 and solve the resulting equation. Those who achieved the correct remainder usually continued to reach the required value of a.

# **Question 2**

This question was testing knowledge of the binomial expansion. It was a very accessible question with a large proportion of candidates scoring all three marks.

Some candidates went directly to the required  $x^7$  whilst others started to write out the full expansion, often stopping at the  $x^7$  term. A roughly equal amount of bracket notation and  ${}^nC_r$  notation was used.

The alternative method of factorising out  $\left(\frac{3}{8}\right)^{12}$  was rarely seen but, when it was, it was generally executed well. The first M mark was gained for combining a correct binomial coefficient  $^{12}C_7$  (or rarely  $^{12}C_5$ ) with  $\left(\frac{3}{8}\right)^5$  and either  $4^7$ ,  $x^7$  or  $(4x)^7$ . Common mistakes here were using the wrong power (usually 4 or 6 instead of 5 for the fractional part) or using the wrong binomial coefficient  $(^{12}C_6, ^{12}C_8, ^{12}C_8,$ 

The first A mark was achieved for obtaining a correct unsimplified term or coefficient. The most common error here was using invisible brackets that were not recovered in further work  $(4x^7)$  instead of  $(4x)^7$  resulting in multiplication by 4 instead of  $4^7$ .

The final A mark was for calculating the  $x^7$  coefficient correctly. This mark was commonly not awarded to candidates who failed to isolate the required term from a list, calculated the term incorrectly or who did not proceed to a value following a correct term.

A high proportion of candidates did not understand the word coefficient, giving an answer of  $96228x^7$  and not 96228; however this was not penalised on this occasion.

#### **Question 3**

This question on the equation of a circle and coordinate geometry proved to be much more challenging for candidates than expected. Typically candidates scored zero, three or six marks; only a quarter were able to successfully score full marks.

In part (a), candidates needed to find the equation of a circle given the coordinates of its centre and a point on the circumference. This was attempted well by the majority of candidates and most achieved all three marks. The most common errors were slips in the substitution of values when calculating the radius, and when writing down the equation of the circle; usually candidates omitted to square the brackets or the radius. A small minority of candidates displayed a lack of understanding of the format for the equation of a circle and instead attempted to use the given coordinates to find the equation of a straight line.

In part (b), candidates needed to find the equation of a horizontal chord of the circle, positioned above the *x*-axis, given its length. This was much less successfully attempted than part (a), with the majority of candidates gaining no marks. Of those who made an attempt, many sketched a diagram, but a significant proportion displayed a poor interpretation of the information given resulting in incorrect methods gaining no marks. The most successful method was using

Pythagoras' Theorem in a geometric approach, calculating the shortest distance from the centre of the circle to the chord MN. However, many of these candidates did not recognise that they needed to add 5 (the y coordinate of the centre) to their value to work out the equation of the chord. The algebraic approach of substituting  $x = 3 \pm 2\sqrt{22}$  into the circle equation to form and solve a three-term quadratic was also fairly common. This generated two possible values for y, but candidates usually selected the correct value for the chord above the x-axis and gained all 3 marks. A few candidates erroneously involved the original point on the circumference, and many made no attempt at all.

#### **Question 4**

This question proved to be a good discriminating question between candidates. A pleasing number scored full marks, however parts (a) and (c) were often poorly completed.

In part (a), candidates struggled to sketch the graph correctly with many having an exponential growth curve. Those candidates who recognised that it was an exponentially decreasing graph mostly gained the first B1 mark as they managed to draw the correct shape in quadrants 1 and 2. A lot of positive reciprocal and quadratic curves (and even straight lines) were seen. Trying to plot several points and joining them was also common. Often the *y*-axis intercept of (0, 5) was omitted, or sometimes it was given as (0, 4) instead. For the final B1 mark the y = 4 horizontal asymptote was often not included or labelled as x = 4. Sometimes candidates thought that the *x*-axis was the asymptote, whilst other times there was no clear asymptote at all.

Generally, candidates found part (b) the most accessible. The majority obtained the correct strip width, usually by considering a single strip, although a few considered the whole table of values by attempting  $h = \frac{b-a}{n}$  to find h but mistakenly used n=6 and not n=5. A pleasing number used the correct structure of the Trapezium Rule, although as has been the case in previous exam sessions, the most common error was bracketing. A few candidates closed the brackets after the initial addition of two terms, then adding the rest afterwards thus gaining no marks. Many failed to give the answer to 2 significant figures but were fortunate the mark scheme said awrt 69, so they scored the mark. Those that simply wrote 70 and did not put the more accurate answer lost the final accuracy mark. Very few candidates attempted to integrate algebraically or used their calculator and wrote down the actual answer.

Candidates were less successful in part (c)(i), as many candidates did not use their answer to part (b), and attempted the trapezium rule again, or attempted to integrate the given expression and apply the limits. Some candidates just subtracted 4 from their answer to part (b) and appeared to misunderstand the nature of the question. Part (c)(ii) proved extremely discriminating. Most candidates were unable to realise that the second integral had the same value with the first and tried to use their calculators to find its value and added to this 69. Some candidates thought that the second integral was -69 and found 0 as their answer.

## **Question 5**

This question was a relatively short and accessible on the topic of geometric sequences and recurrence relationships. Many candidates found this particularly challenging, however, and it was extremely rare for full marks to be scored.

In part (i), candidates were either able to recognise that the terms of the sequence formed a geometric sequence and consequently found the sum to infinity, or attempted to write out the separate terms. Many candidates correctly used the sum to infinity formula, however a number resorted to using the summation button on their calculator which, even if 2 was found, no marks were scored. Some candidates did write out the first three terms which demonstrated an understanding of sigma notation and this could score the first mark. Those who did attempt the sum to infinity often used a = 6 instead of a = 1.5 in the sum to infinity formula.

In part (ii), most candidates were able to score the method mark in (a). The vast majority of candidates were able to achieve the  $2^{nd}$ ,  $3^{rd}$  and  $4^{th}$  terms of the sequence, but they often struggled with a suitable explanation or conclusion that the sequence was periodic. It was rare that reasoning was given such as  $u_1 = u_4$  but it was condoned on this occasion that stating the sequence was periodic could score the second mark, provided the correct terms were found.

Part (b) provided to be one of the toughest marks to be scored by candidates. Most did not understand what "order" meant in terms of periodic sequences and just stated the order of the terms rather than stating that the order was 3.

In part (c), it was pleasing to see many candidates correctly use their part (a) to find the sum of the first 70 terms. However, a number either forgot to add 3 for the  $70^{th}$  term and just found the sum of the first 69 terms. A few tried to use  $\frac{70}{3}\left(3+0+\frac{3}{2}\right)$ . Other candidates attempted a sum of an arithmetic sequence or geometric sequence which did not score any marks.

#### **Question 6**

While many candidates demonstrated some knowledge of logarithms, only the strongest candidates were able to achieve full marks in this question losing especially the final B1 mark in part(b)(ii).

Part (a) was well answered, as most candidates demonstrated a good understanding of the laws of logs by writing  $2\log_4(x+3)$  as  $\log_4(x+3)^2$ , or  $\frac{1}{2}$  as  $\log_4 2$ , and then correctly combined at least two of the original terms. Most candidates were then able to write a correct intermediate equation not involving logarithms. For weaker candidates, the most common error was first applying the product rule on the left-hand side, then the power rule and getting  $2\log_4 x(x+3) = \log_4(x^2+3x)^2$ . Other errors were adding instead of multiplying or removing

all logs to get e.g.  $2(x+3)+x=(4x+2)+\frac{1}{2}$  or splitting logs such as  $\log_4(x+3)=\log_4x+\log_43$ . Sometimes the final A1 mark was not gained because candidates did not show enough working before reaching the final answer such as not expanding  $(x+3)^2$  first. Less rare was the incorrect application of the subtraction log rule such as writing  $\frac{\log ...}{\log ...}$  and cancelling the logs from the top and the bottom which prevented the final two marks from being scored.

In part (b)(i) a majority of candidates realised that if x = -1 is a root, then (x+1) is a factor of the cubic equation and mostly used long division. Some used the inspection method to find the correct quadratic factor and most of them were successful in doing this. Many candidates used the quadratic formula and a few used the completing square method to solve the quadratic equation to find the other roots. A significant number used their calculators and directly wrote the correct exact roots gaining full marks as calculators were allowed at this stage of the question. A small number of candidates tried to obtain a quadratic by writing  $x(x^2 + 6x + 1) = 4$  gaining no marks. A few candidates relied on calculator technology without finding the quadratic factor of the cubic expression and only stated the decimal approximations of the two roots as well as x = -1; this scored no marks in this part.

In part (b)(ii), most candidates lost the B1 mark in this part by writing x = -1 and both positive and negative roots obtained in part (i) without considering whether all these values were valid. Only a minority realised that logs are defined for positive values only and selected the positive root only.

## **Question 7**

This question was very accessible on the top of arithmetic and geometric sequences. Whilst it was a different context, the style of question should have been familiar to candidates who had prepared thoroughly for the exam and most candidates performed well.

In part (a), the vast majority of candidates were able to use the formula for an arithmetic progression successfully to achieve a value for d and then go onto calculate the fourth term correctly. Rounding was not done correctly by a large proportion of candidates and awrt and isw proved very helpful to many of them. Furthermore, a number of candidates lost the final A mark due to early approximation of their d value. Where candidates did not achieve a correct answer, due to an incorrect method, this was generally due to two reasons: some candidates used n instead of n-1 in their calculations, meaning they could at least gain the second M mark for finding the fourth term. Others misinterpreted the 4000 to be the sum and thus incorrectly used the sum formula resulting in no marks. It was rare to see candidates mistaking this part for a GP.

Part (b) was well answered by most candidates, but some failed to find the 11th root to achieve a valid value of r. A few candidates did not use the GP formula correctly. Many used the approximated value of r to further their workings with some losing the final mark. Some were unable to make r the subject of the formula and find the correct value for r. A few misread the question and found the fourth term rather than the second that was required.

In part (c), a significant number of candidates scored only one mark on this part by failing to find the difference between the sum of the AP and GP. The first method mark, for finding the sum of either the AP or GP, was achieved by most candidates but the second method mark were lost as many did not find the difference. A lot of candidates lost the accuracy mark here by using rounded values in their calculations. Whilst some candidates were confused with the two types of sequence in part (a) and part (b) they were still able to access full marks in part (c).

### **Question 8**

This question enabled candidates to demonstrate their knowledge and ability to apply two different styles of proof. A wide range of marks was seen as fully complete answers proved a challenge.

In part (i), almost all candidates were able to find a counter example to prove the statement was false. Most of these choose to use n = 6 leading to 55. Some candidates then did not conclude that "55" was not a prime number or that  $55 = 5 \times 11$  which meant that the original statement was false. A small number of candidates were unaware of which numbers were natural numbers; some used negative numbers and several used zero.

In part (ii) many candidates failed to notice or take heed of the question which said "use algebra to prove..." As a result candidates tried to use various numbers to show that the expression was not a multiple of 4. Those that used algebra tended to use either 2p and 2p+1 for even and odd numbers, or the less common 4k, 4k+1, 4k+2 and 4k+3. These were usually correctly substituted into the expression to gain both method marks in this part. The A marks were for concluding that the resultant expressions were not a factor of 4; those that chose to use 4k etc were often better at going on to partially factorise out 4 from the expression to demonstrate that the expression was not a multiple of 4, however conclusions were on the whole minimal and an overall conclusion that they had completed the proof for all natural numbers was often not given resulting in the loss of the final mark.

## **Question 9**

Overall, this question proved harder than anticipated for candidates to gain full marks as often even the more competent candidates did not heed the comment about the use of calculator technology, and to show detailed reasoning.

Part (i) required candidates to manipulate an equation using trigonometric identities to form a quadratic in  $\cos\theta$  and solve to find  $\theta$  in a given range. Almost all candidates were able to achieve the first mark with involved using  $\tan\theta = \frac{\sin\theta}{\cos\theta}$ . The majority continued to then use  $\sin^2\theta = 1 - \cos^2\theta$  to form a quadratic in  $\cos\theta$ . Many also continued to solve the quadratic and to find the arccos of the answer to obtain the first solution. The second solution proved more elusive with many not doing it or using 180+the original angle. On the whole, however, the

Part (ii) required candidates to understand the transformation of functions, in this case the sine function. This part of the question had a much wider distribution of marks and showing detailed reasoning for each part was often missing. Correct answers with no working or insufficient working were common; this resulted in only two marks being scored.

majority of candidates did well with this part of the question.

The first requirement of this part was to state the value of A; most achieved the correct answer of 5 but several had 7 as they had not appreciated the vertical translation by 2. The second part required them to find the first positive minimum point of the graph; often those that showed working ended up with  $-\frac{\pi}{16}$  and many did not sure any working for this part of the question leading to only the final mark being scored.

The final section to this part was to find the fourth time that the graph crossed the positive x-axis. Many candidates achieved  $\sin\left(2\theta - \frac{3}{8}\pi\right) = "-\frac{2}{5}"$  but then often jumped to either the final answer or to another of the other values where the graph crossed the x-axis, however with no detailed working this could only score the first mark. Whilst this was definitely one of the most complex parts in the entire paper, the first few marks were still accessible, though that said, this was left blank by many. Only the most established mathematicians were able to score full marks

in this part, but a good number of those who attempted it were able to score at least one or two

marks; it was particularly the last step which proved too challenging for most candidates, with the most common incorrect answer being 0.38.

#### **Question 10**

Candidates should note that when the question demands the use of algebraic integration, they must show all their workings and not rely on the calculator for their answers.

In part (a), many candidates were able to differentiate at least one term correctly. Most were able to deal with the fractional power and differentiate the second term, too, hence earning the first two marks for this question. However, many were unable to handle the algebra involved in processing the indices in their  $\frac{dy}{dx}$ , so they lost the final two marks. A few substituted the

value x = 9 in their  $\frac{dy}{dx}$  to verify that this was a solution, however, a small number omitted to give a conclusion at the end.

The demand of part (b) was to use algebraic integration which far too many candidates failed to appreciate by using their calculators throughout, thus forfeiting the marks. However, most were able to integrate the expression correctly and obtained the first two marks. The connection between integration and area was often absent and many did not understand which area was required. Often candidates failed to consider the area of the rectangle or did not subtract this area. Some of those who correctly proceeded to find the area R did not give exact answers, as required by the question, and lost the final mark. Some candidates also did not show the limits substituted in their integrated expression (or evidence of having done this) which resulted in the final two marks not being scored.